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The He–McKellar–Wilkens effect for spin one particles in non-commutative quantum mechanics

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Abstract
The He–McKellar–Wilkens (HMW) effect for spin one neutral particles in non-commutative quantum mechanics is studied. By solving the Kemmer-like equations on the non-commutative (NC) space and non-commutative phase space, we obtain the topological He–McKellar–Wilkens phase on the NC space and NC phase space respectively, where the additional terms related to the space–space and momentum–momentum non-commutativity are given explicitly.

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1. Introduction

The study of physics effects on the non-commutative space has attracted much attention in recent years. Because the effects of the space non-commutativity may become significant at the very high (Tev or above) energy scale. Besides the field theory, there are many papers devoted to the study of various aspects of quantum mechanics on NC space with a usual time coordinate [1–10]. For example, the Aharonov–Bohm phase on the NC space and NC phase space has been studied in [2–4]. The Aharonov–Casher phase for a spin half and spin-1 particle on the NC space and NC phase space has been studied in [5–8]. The equivalence of the Aharonov–Bohm and Aharonov–Casher effects was studied in the relativistic case for spin half particles in [11]. The He–McKellar–Wilkens (HMW) effect on commutative space was firstly discussed in 1993 by He and Meckellar [12] and a year later, independently by Wilkens [13]. The HMW effect corresponds to a topological phase related to a neutral particle with non-zero electric dipole moment moving in a pure magnetic field, and in 1998, Dowling, Williams and

Franson pointed out that the HMW effect can be partially tested using metastable hydrogen atoms [14]. The HMW phase for a spin half particle on the NC space and NC phase space has been studied in [9], but there is no discussion about the HMW effect for spin one neutral particles in the literature, so in this paper we study the HMW effect of spin one particles.

Let us first review some basic concepts of NC quantum mechanics. In the usual commutative case the algebra of observables $\mathcal{A}$ is generated by operators x and p satisfying the standard commutation relations (we take $\hbar = c = 1$)

$$[x_\nu, x_\lambda] = [p_\nu, p_\lambda] = 0, \qquad [x_\nu, p_\lambda] = i\delta_{\nu\lambda}. \tag{1}$$

In the NC case the commutators of the generators of the algebra of observables $\hat{\mathcal{A}}$ are replaced by the deformed ones, i.e. the deformed $\hat{x}$ and $\hat{p}$ satisfy [10]

$$[\hat{x}_\nu, \hat{x}_\lambda] = i\Theta_{\nu\lambda}, \qquad [\hat{p}_\nu, \hat{p}_\lambda] = i\bar{\Theta}_{\nu\lambda}, \qquad [\hat{x}_\nu, \hat{p}_\lambda] = i\delta_{\nu\lambda}. \tag{2}$$

The NC variables can be expressed (up to some singular cases) as

$$\hat{x}_\nu = \alpha x_\nu - \frac{1}{2\alpha}\Theta_{\nu\lambda}p^\lambda, \qquad \hat{p}_\nu = \alpha p_\nu + \frac{1}{2\alpha}\bar{\Theta}_{\nu\lambda}x^\lambda. \tag{3}$$

Therefore, the algebras $\mathcal{A}$ and $\hat{\mathcal{A}}$ are the same. The pure states in both cases are given as wavefunctions $\psi(x)$ in the Hilbert space. In the NC case one has the following action (up to terms linear in θ's): $\hat{x}\psi(x) = (\alpha x - (1/2\alpha)\theta\wedge p)\psi(x)$, $\hat{p}\psi(x) = (\alpha p + (1/2\alpha)\bar{\theta}\wedge x)\psi(x)$. The difference between usual quantum mechanics and NC quantum mechanics are visible only after the choice of the Hamiltonian. If the usual Hamiltonian is given as $H = H(p, x)$, then the standard choice for the NC Hamiltonian $\hat{H} = H(\hat{p}, \hat{x})$ determines the NC interpretation of the NC quantum mechanics system in question. The corresponding Schrödinger equation contains the action on a wavefunction described above. Therefore, there are no differences at the level of kinematics between the usual quantum mechanics and the NC quantum mechanics; the difference between them is specified by dynamics.

In NC quantum mechanics, the Schrödinger equation, as we know, can be written as $H(p, x) * \psi(p, x) = E\psi(p, x)$, from the discussion above, this NC Schrödinger equation can be equivalently written by $H(\hat{p}, \hat{x})\psi(p, x) = E\psi(p, x)$, i.e. the star product can be changed into an ordinary product by making shifts [10] $x \to \hat{x}$, $p \to \hat{p}$. In the case of this paper, the Hamiltonian also depends on the dual of the electromagnetic tensor $\tilde{F}$, so, when the star product is replaced by the usual product in the Schrödinger equation, the $\tilde{F}$ should also be shifted as,

$$\tilde{F}_{\nu\lambda} \to \hat{\tilde{\mathcal{F}}}_{\nu\lambda} = \alpha\tilde{F}_{\nu\lambda} + \frac{1}{2\alpha}\Theta^{\rho\sigma}p_\rho\partial_\sigma\tilde{F}_{\nu\lambda}. \tag{4}$$

When only space–space non-commutativity is considered we call it the NC space, when both space–space and momentum–momentum non-commutativity are considered we call it the NC phase space. On NC space, $\bar{\Theta} = 0$, and it leads to $\alpha = 1$, the equations (3) and (4) are reduced to the well-known results [2]:

$$\hat{x}_\nu = x_\nu - \tfrac{1}{2}\Theta_{\nu\lambda}p^\lambda, \qquad \hat{p}_\nu = p_\nu, \tag{5}$$

$$\tilde{F}_{\nu\lambda} \to \hat{\tilde{F}}_{\nu\lambda} = \tilde{F}_{\nu\lambda} + \tfrac{1}{2}\Theta^{\rho\sigma}p_\rho\partial_\sigma\tilde{F}_{\nu\lambda}. \tag{6}$$

In this paper, first we discuss He–McKellar–Wilkens effect for a spin-1 neutral particle with non-zero electric dipole moment moving in the magnetic field on commutative space. Then we study the He–McKellar–Wilkens effect on non-commutative space and give a generalized formula of the HMW phase. We also give a generalized formula of the HMW phase on the non-commutative phase space. Conclusion remarks are given in the last section.

2. The HMW effect for spin one particles in quantum mechanics

In a similar way as in Aharonov–Bohm, Aharonov–Casher topological effects, the He–McKellar–Wilkens effect can also be studied in 2 + 1 dimension. The ordinary configuration for HWM effect is a neutral particle with nonzero electric dipole moment μ_e moves in a pure magnetic field produced by an infinitely long filament which is uniformly charged with magnetic charge (monopoles) and the filament is perpendicular to the plane, let us say the x–y plane, then the problem can be treated in 2 + 1 spacetime. We use the conventions $g_{\mu\nu} = \mathrm{diag}(1, -1, -1)$.

The Dirac-like equation of a spin one neutral particle with electric dipole μ_e moving in the electromagnetic field is called Kemmer equation and is given by [18]

$$\left(i\beta^{\nu}\partial_{\nu} - \tfrac{1}{2}\mu_e S_{\lambda\rho}\tilde{F}^{\lambda\rho} - m\right)\phi = 0, \qquad (7)$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ is the 3 + 1-dimensional dual of the electromagnetic field tensor. In 2 + 1 dimensions, its explicit form is

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & -B^1 & -B^2 \\ B^1 & 0 & -E^3 \\ B^2 & E^3 & 0 \end{pmatrix},$$

the 10 × 10 matrices β_{ν} are generalization of the 4 × 4 Dirac gamma matrices, and it can be chosen as follows [17–20]

$$\beta^0 = \begin{pmatrix} \widehat{O} & \widehat{O} & I & o^{\dagger} \\ \widehat{O} & \widehat{O} & \widehat{O} & o^{\dagger} \\ I & \widehat{O} & \widehat{O} & o^{\dagger} \\ o & o & o & 0 \end{pmatrix}, \qquad \beta^j = \begin{pmatrix} \widehat{O} & \widehat{O} & \widehat{O} & -iK^{j\dagger} \\ \widehat{O} & \widehat{O} & S^j & o^{\dagger} \\ \widehat{O} & -S^j & \widehat{O} & o^{\dagger} \\ -iK^j & o & o & 0 \end{pmatrix},$$

with $j = 1, 2, 3$. The elements of the 10 × 10 matrices β_{ν} are given by the matrices

$$\widehat{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$S^1 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S^2 = i\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \qquad S^3 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$o = (0 \quad 0 \quad 0), \qquad K^1 = (1 \quad 0 \quad 0), \qquad K^2 = (0 \quad 1 \quad 0), \qquad K^3 = (0 \quad 0 \quad 1).$$

The above β matrices satisfy the following relation

$$\beta_{\nu}\beta_{\lambda}\beta_{\rho} + \beta_{\rho}\beta_{\lambda}\beta_{\nu} = \beta_{\nu}g_{\lambda\rho} + \beta_{\rho}g_{\nu\lambda}. \qquad (8)$$

And other algebraic properties of the Kemmer β-matrices were given in [18]. $S_{\lambda\rho}$ is the Dirac $\sigma_{\lambda\rho}$ like spin operator, which can be defined as

$$S_{\lambda\rho} = \tfrac{1}{2}(\beta_{\lambda}\beta_{\rho} - \beta_{\rho}\beta_{\lambda}). \qquad (9)$$

The solution of the Kemmer equation can be written in the following form

$$\phi = \mathrm{e}^{-\mathrm{i}\xi_3\int^{r}\mathbf{a}\cdot\mathbf{dr}}\phi_0, \qquad (10)$$

where ϕ_0 is a solution of the free Kemmer equation; the spin one pseudo-vector operator ξ_ν in (10) is defined as

$$\xi_\nu = \frac{\mathrm{i}}{2}\varepsilon_{\nu\lambda\rho\sigma}\beta^\lambda\beta^\rho\beta^\sigma, \tag{11}$$

where $\varepsilon_{\nu\lambda\rho\sigma}$ is the Levi-Civita symbol in four dimensions. Now we need to find the explicit form of the vector $\mathbf{a}$ in (10). To do this, first we write the free Kemmer equation for ϕ_0 in terms of ϕ

$$(i\beta^\nu\partial_\nu - m)\,\mathrm{e}^{i\xi_3\int^r \mathbf{a}\cdot\mathbf{dr}}\phi = 0 \tag{12}$$

We impose the following two conditions in order to have the equivalence of (7) and (12)

$$\mathrm{e}^{-\mathrm{i}\xi_3\int^r \mathbf{a}\cdot\mathbf{dr}}\beta^\nu e^{i\xi_3\int^r \mathbf{a}\cdot\mathbf{dr}} = \beta^\nu, \tag{13}$$

and

$$\beta^\nu\xi_3 a_\nu\phi = \tfrac{1}{2}\mu_e S_{\lambda\rho}\tilde{F}^{\lambda\rho}\phi = \mu_e S_{0l}\tilde{F}^{0l}\phi. \tag{14}$$

By comparing (13) with the Baker–Housdorf formula

$$e^{-\mathrm{i}\lambda\xi_3}\beta^\nu e^{i\lambda\xi_3} = \beta^\nu + \wp(-i\lambda)[\xi_3,\beta^\nu] + \frac{1}{2!}\wp(-i\lambda)^2[\xi_3,[\xi_3,\beta^\nu]]\ldots, \tag{15}$$

we get, $[\xi_3,\beta^\nu] = 0$, where $\wp$ stands for path ordering of the integral in the phase. If $\nu \neq 3$ this commutation relation is automatically satisfied, however, for $\nu = 3$, by using (8) and (11), we find that the commutator does not vanish. Thus, to satisfy the first condition we restrict the particle in the x–y plane, that is, $B_z = 0$. In particular $\partial_3\phi = 0$ and $a_3 = 0$. From (14), by using (8), (9) and (11), one obtains

$$a_l = 2\mu_e\varepsilon_{lk}B_k, \qquad l,k = 1,2. \tag{16}$$

Thus the HMW phase for a neutral spin one particle moving in a 2 + 1 spacetime under the influence of a pure magnetic field produced by an infinitely long filament which is uniformly charged with magnetic monopoles is given by

$$\phi_{\mathrm{HMW}} = \xi_3\oint \mathbf{a}\cdot\mathbf{dr} = 2\mu_e\xi_3\varepsilon^{lk}\oint B_l\,\mathrm{d}x_k = 2\mu_e\xi_3\oint(-B_1\,\mathrm{d}x_2 + B_2\,\mathrm{d}x_1). \tag{17}$$

The above equation can also be written as

$$\phi_{\mathrm{HMW}} = \xi_3\oint \mathbf{a}\cdot\mathbf{dr} = \xi_3\int_S(\nabla\times\mathbf{a})\cdot\mathrm{d}\mathbf{S} = 2\mu_e\xi_3\int_S(\nabla\cdot\mathbf{B})\,\mathrm{d}S = 2\mu_e\xi_3\lambda_m, \tag{18}$$

where λ_m is the magnetic charge density of the filament. This spin one the HMW phase is also purely quantum mechanical effect and has no classical interpretation. One may note that the HMW phase for spin one particles is exactly the same as those for spin half, except that the spin operator and spinor have changed. The factor of two shows that the phase is twice that accumulated by a spin half particle with the same electric dipole moment, in the same magnetic field.

3. HMW effect for spin one particles in non-commutative quantum mechanics

In this section we study HMW effect for spin one particles both on the NC space and NC phase space. By replacing the usual product in (7) with a star product (Moyal–Weyl product), the Kemmer equation for a spin one neutral particle with a electric dipole moment μ_e, on NC space, can be written as

$$\left(i\beta^\nu\partial_\nu - \tfrac{1}{2}\mu_e S_{\lambda\rho}\tilde{F}^{\lambda\rho} - m\right)*\phi = 0, \tag{19}$$

By (6), we replace the star product in (19) with the ordinary product, then the Kemmer equation on the NC space has the form

$$\left(i\beta^{\nu}\partial_{\nu} - \tfrac{1}{2}\mu_e S_{\lambda\rho}\hat{\tilde{F}}^{\lambda\rho} - m\right)\phi = 0. \tag{20}$$

In a similar way as the commuting space, the solution of the above equation can also be written as

$$\phi = \mathrm{e}^{-\mathrm{i}\xi_3\int^r \hat{\mathbf{a}}\cdot\mathrm{d}\mathbf{r}}\phi_0. \tag{21}$$

To determine $\hat{\mathbf{a}}$ we write the free Kemmer equation as

$$(i\beta^{\nu}\partial_{\nu} - m)\,\mathrm{e}^{i\xi_3\int^r \hat{\mathbf{a}}\cdot\mathrm{d}\mathbf{r}}\phi = 0. \tag{22}$$

The equivalence of (20) and (22) gives the following two conditions

$$\mathrm{e}^{-\mathrm{i}\xi_3\int^r \hat{\mathbf{a}}\cdot\mathrm{d}\mathbf{r}}\beta^{\nu}\mathrm{e}^{i\xi_3\int^r \hat{\mathbf{a}}\cdot\mathrm{d}\mathbf{r}} = \beta^{\nu} \tag{23}$$

and

$$\beta^{\nu}\xi_3\hat{a}_{\nu}\phi = \tfrac{1}{2}\mu_e S^{\lambda\rho}\hat{\tilde{F}}_{\lambda\rho}\phi = \mu_e S^{0l}\hat{\tilde{F}}_{0l}\phi. \tag{24}$$

By using (15), the first condition (23) implies that, $[\xi_3, \beta^{\nu}] = 0$. If $\nu \neq 3$ then this commutation relation is automatically satisfied; however, for $\nu = 3$, by using (8) and (11), one finds that the commutator does not vanish. Therefore, in order to fulfil the first condition we restrict the particle in 2 + 1 spacetime. In particular $\partial_3\phi = 0$ and $\hat{a}_3 = 0$. From (24), and by using (8), (9) and (11), we obtain

$$\begin{aligned} \hat{a}_1 &= 2\mu_e\hat{\tilde{F}}_{02} = 2\mu_e\tilde{F}_{02} + 2\mu_e\tfrac{1}{2}\Theta^{ij}p_i\partial_j\tilde{F}_{02} = 2\mu_e B_2 + \mu_e\theta\varepsilon^{ij}p_i\partial_j B_2 \\ \hat{a}_2 &= -2\mu_e\hat{\tilde{F}}_{01} = -2\mu_e\tilde{F}_{01} - 2\mu_e\tfrac{1}{2}\Theta^{ij}p_i\partial_j\tilde{F}_{01} = -2\mu_e B_1 + \mu_e\theta\varepsilon^{ij}p_i\partial_j B_1 \end{aligned} \tag{25}$$

with $\Theta^{ij} = \theta\epsilon^{ij}, \Theta^{0\mu} = \Theta^{\mu 0} = 0$; $\epsilon^{ij} = -\epsilon^{ji}, \epsilon^{12} = +1$. Thus the HMW phase for a neutral spin one particle moving in a 2 + 1 non-commutative space under the influence of a pure magnetic field produced by an infinitely long filament which is uniformly charged with magnetic monopoles, is

$$\hat{\phi}_{\mathrm{HMW}} = \xi_3\oint\hat{\mathbf{a}}\cdot\mathrm{d}\mathbf{r} = 2\mu_e\xi_3\varepsilon^{lk}\oint B_l\,\mathrm{d}x_k + \mu_e\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint p_i\partial_j B_l\,\mathrm{d}x_k. \tag{26}$$

In a similar way as in spin half case [9], the momentum on NC space for a spin-1 neutral particle can also be written as

$$p_i = mv_i + (\vec{B}\times\vec{\mu})_i + \mathcal{O}(\theta), \tag{27}$$

where $\vec{\mu} = 2\mu_e\vec{S}$, and $\vec{S}$ is the spin operator of the spin one. By inserting (27) into (26), we have

$$\hat{\phi}_{\mathrm{HMW}} = \phi_{\mathrm{HMW}} + \delta\phi_{\mathrm{NCS}}, \tag{28}$$

where ϕ_{HMW} is the HMW phase (17) on the commuting space; the additional phase $\delta\phi_{\mathrm{NCS}}$, related to the non-commutativity of space, is given by

$$\delta\phi_{\mathrm{NCS}} = \mu_e\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint[k_i - (\vec{\mu}\times\vec{B})_i]\partial_j B_l\,\mathrm{d}x_k \tag{29}$$

where $k_i = mv_i$ is the wave number; the ξ_3 in the phase represents the spin degrees of freedom. If the spin of the neutral particle along the z direction, namely, $\vec{\mu} = 2\mu_e s_3\hat{\vec{k}}$, then the above equation takes the form

$$\delta\phi_{\mathrm{NCS}} = \mu_e\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint[k_i - 2\mu_e s_3(\hat{\vec{k}}\times\vec{B})_i]\partial_j B_l\,\mathrm{d}x_k, \tag{30}$$

where $\hat{\vec{k}}$ is a unit vector in the z direction; $s_3 = 1, 0, -1$.

Now we discuss the HMW phase on the NC phase space. From (3), (4) and (19), the Kemmer equation for HMW problem on the NC phase space has the form

$$\left(-\beta^{\nu}\hat{p}_{\nu} - \tfrac{1}{2}\mu_e S_{\lambda\rho}\hat{\hat{\mathcal{F}}}^{\lambda\rho} - m\right)\phi = 0. \tag{31}$$

Because $\alpha \neq 0$, the above equation can be written as

$$\left(-\beta^{\nu}p_{\nu} - \frac{1}{2\alpha^2}\beta^{\nu}\bar{\Theta}_{\nu\lambda}x^{\lambda} - \frac{1}{2}\mu_e S_{\lambda\rho}(\tilde{F}^{\lambda\rho} + \frac{1}{2\alpha^2}\Theta^{\sigma\tau}p_{\sigma}\partial_{\tau}\tilde{F}^{\lambda\rho}) - m'\right)\phi = 0. \tag{32}$$

where $m' = m/\alpha$. We write the above equation in the following form

$$(-\beta^{\nu}p_{\nu} - m')\,\mathrm{e}^{\frac{\mathrm{i}}{2\alpha^2}\int^{r}\bar{\Theta}_{\nu\lambda}x^{\lambda}\,\mathrm{d}x^{\nu} + i\xi_3\int^{r}\hat{\mathbf{a}}'\cdot d\mathbf{r}}\phi = 0. \tag{33}$$

To have the equivalence of (32) and (33), we impose the following two conditions

$$\mathrm{e}^{-\mathrm{i}\xi_3\int^{r}\hat{\mathbf{a}}'\cdot\mathrm{d}\mathbf{r}}\beta^{\nu}\mathrm{e}^{i\xi_3\int^{r}\hat{\mathbf{a}}'\cdot\mathrm{d}\mathbf{r}} = \beta^{\nu}, \tag{34}$$

and

$$-\beta^{\nu}\xi_3\hat{a}'_{\nu}\phi = \frac{1}{2\alpha}\mu_e S_{\lambda\rho}\hat{\hat{F}}^{\lambda\rho}\phi = \frac{\mu_e}{\alpha}S_{0l}\hat{\hat{F}}^{0l}\phi. \tag{35}$$

In an analogous way as in NC space, from (34) and (35) one obtains

$$\begin{aligned}
\hat{a}'_1 &= \frac{2\mu_e}{\alpha}\hat{\hat{F}}^{02} = 2\mu_e\tilde{F}^{02} + 2\mu_e\frac{1}{2\alpha^2}\Theta^{ij}p_i\partial_j\tilde{F}^{02} = 2\mu_e B_2 + \frac{\mu_e\theta}{\alpha^2}\varepsilon^{ij}p_i\partial_j B_2,\\
\hat{a}'_2 &= -\frac{2\mu_e}{\alpha}\hat{\hat{F}}^{01} = -2\mu_e\tilde{F}^{01} - 2\mu_e\frac{1}{2\alpha^2}\Theta^{ij}p_i\partial_j\tilde{F}^{01} = -2\mu_e B_1 - \frac{\mu_e\theta}{\alpha^2}\theta\varepsilon^{ij}p_i\partial_j B_1,\\
\hat{a}'_3 &= 0.
\end{aligned} \tag{36}$$

Thus the HMW phase for a neutral spin one particle moving in a 2 + 1 non-commutative phase space under the influence of a pure magnetic field produced by an infinitely long filament, which is uniformly charged with magnetic monopoles, and which is perpendicular to the plane, is given by

$$\begin{aligned}
\hat{\phi}_{\mathrm{HMW}} &= \frac{1}{2\alpha^2}\oint\bar{\Theta}_{\nu\lambda}x^{\lambda}\,\mathrm{d}x^{\nu} + \xi_3\oint\hat{\mathbf{a}}'\cdot\mathrm{d}\mathbf{r}\\
&= \frac{\theta}{2\alpha^2}\oint\varepsilon^{ij}x_j\,\mathrm{d}x_i + 2\mu_e\xi_3\varepsilon^{lk}\oint B_l\,\mathrm{d}x_k + \mu_e\xi_3\frac{\theta}{\alpha^2}\varepsilon^{ij}\varepsilon^{lk}\oint p_i\partial_j B_l\,\mathrm{d}x_k
\end{aligned} \tag{37}$$

By using $p_i = k'_i + (\vec{B}\times\vec{\mu})_i + \mathcal{O}(\theta)$, and $k'_i = m'_i v_i$, $\vec{\mu} = 2\mu_e\vec{S}$, one obtains

$$\hat{\varphi}_{\mathrm{HMW}} = \phi_{\mathrm{HMW}} + \delta\phi_{\mathrm{NCS}} + \delta\phi_{\mathrm{NCPS}}, \tag{38}$$

where ϕ_{HMW} is the HMW phase (17) on the commuting space; $\delta\phi_{\mathrm{NCS}}$ is the space–space non-commuting contribution to the HMW phase (17) , and its explicit form is given in (29); the last term $\delta\phi_{\mathrm{NCPS}}$ is the momentum–momentum non-commuting contribution to the HMW phase, and it has the form

$$\begin{aligned}
\delta\phi_{\mathrm{NCPS}} &= \frac{\bar{\theta}}{2\alpha^2}\oint\varepsilon^{ij}x_j\,\mathrm{d}x_i + \left(\frac{1}{\alpha^2} - 1\right)\mu_e\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint k'_i\partial_j B_l\,\mathrm{d}x_k\\
&\quad - \left(\frac{1}{\alpha^2} - 1\right)\mu_e\xi_3\theta\varepsilon^{ij}\varepsilon^{lk}\oint(\vec{\mu}\times\vec{B})_i\partial_j B_l\,\mathrm{d}x_k,
\end{aligned} \tag{39}$$

which represents the non-commutativity of the momenta. The first term in (39) comes from the momentum–momentum non-commutativity; the second term is a velocity dependent correction and does not have the topological properties of the commutative HMW effect and could modify the phase shift; the third term is a correction to the vortex and does not contribute to the line spectrum. In two-dimensional non-commutative plane, $\bar{\Theta}_{ij} = \bar{\theta}\epsilon_{ij}$, and the two NC parameters θ and $\bar{\theta}$ are related by $\bar{\theta} = 4\alpha^2(1-\alpha^2)/\theta$ [10]. When $\alpha = 1$, which will lead to $\bar{\theta}_{ij} = 0$, then the HMW phase on the NC phase space will return to the HMW phase on NC space, i.e. $\delta\phi_{\mathrm{NCPS}} = 0$ and equation (38) will change to equation (28).

4. Conclusion remarks

There are two methods, namely, star product and shift method, to study physical effects on the NC space and NC phase space. In this paper, first study the HMW effect in quantum mechanics. Then by using the shift method we give the NC space corrections to the topological phase of the HMW effect for a spin one neutral particle. Furthermore, by considering the momentum–momentum non-commutativity we obtain the NC phase space corrections to the topological phase of the HMW effect for a spin one neutral particle. We note that the corrections (29) and (39) to the topological phase (17) or (18) of the HMW effect for a spin one neutral particle both on the NC space and NC phase space can be obtained from spin half corrections [9] through the replacement $\frac{1}{2}\gamma^0\sigma^{12} \longrightarrow \xi_3$. One may conclude that, apart from the spin operators, the NC HMW phase for a higher spin neutral particle is the same as those for spin half and spin one case in non-commutative quantum mechanics.

The method we use in this paper may also be employed to other physics problem on the NC space and NC phase space.

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